Technical Comments

Comment on "Relationship Between Kane's Equations and the Gibbs-Appell Equations"

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DESLOGE[1] purports to show that "Kane's equations are simply a particular form of the Gibbs...equations...." Now, any valid method for formulating dynamical equations can be shown to be intimately related to any other. Hence, it is a foregone conclusion that Kane's equations can be obtained with the aid of Newton's and that Gibbs' can be deducted from both; in fact, any one of these principles can be regarded, with equal justice, as a "particular form" of any of the others. Hence, this aspect of Desloge's Note has no merit. However, the Note can give rise to two questions of real substance, namely, do Kane's and Gibbs' methods differ from each other significantly, and which one is better suited for the actual formulating of equations of motion for complex multibody systems? An experienced dynamicist who has used both Kane's method and Gibbs' to formulate, in all detail, explicit dynamical equations for truly complex systems can answer both questions immediately: The two methods differ from each other significantly, and Kane's method is superior. Let us examine these claims briefly.

The basis for Kane's method is his observation that, for a dynamical system S possessing n degrees of freedom, the inertial angular velocity ω of any rigid body belonging to S can always be expressed as

$$\omega \neq \sum_{r=1}^{n} \omega_r u_r + \omega_t \tag{1}$$

and the inertial velocity v of any point of S can always be written as

$$v = \sum_{r=1}^{n} v_r u_r + v_t \tag{2}$$

where $u_1,\ldots,u_n$, called *generalized speeds*, are "speed-like" quantities, usually scalar components of physically important angular velocity and linear velocity vectors, chosen by the analyst to characterize the motion of S. Kane's considerable contribution to dynamics is that he has brought to light the fact that the vectors $\omega_1,\ldots\omega_n$ [see Eq. (1)], called *partial angular velocities*, and the vectors $v_1,\ldots v_n$ [see Eq. (2)], called *partial velocities*, are important quantities, directly useful in engineering analysis. He has pointed out that ω_r and v_r, found by simply inspecting expressions for angular velocity and linear velocity vectors, can be dot-multiplied with familiar, readily available quantities, such as accelerations, forces, etc., to produce first-order differential equations of motion having the simplest possible form and thus being ideally suited for numerical solution. The use of partial angular velocities and partial velocities enables one to formulate *linearized* equations of motion in a particularly efficient way. It eliminates the need to introduce Lagrange multipliers in connection with nonholonomic systems. It frees one from the need to invoke virtual work concepts. It leads to the automatic elimination of nonworking constraint forces, but permits the easy evaluation of any such forces that may be of interest in a particular situation. This is an incomplete list of the benefits one can derive from using Kane's method. Realizing all this, many practicing engineers share the view that this method and its exposition rank among the greatest contributions to practical dynamic analysis in our time.

Gibbs' method, not involving ω_r and v_r, instead necessitates, first, forming an extraneous function, called the Gibbs function, that contains squares of accelerations as well as complicated combinations of dot-products and cross-products of angular velocities, angular accelerations, and inertia dyadics. Next, one must form partial derivatives of the Gibbs function in order to generate contributions to the equations of motion. This procedure is particularly arduous in connection with rigid-body contributions, forcing one to derive again and again expressions which in Kane's method are available from the outset. Thus, Desloge has it backwards when he asserts that "the Gibbs...equations are generated quite simply and elegantly from a single scalar quantity..., whereas vector quantities are required to generate Kane's equations." It is precisely the effective use of appropriate vector quantities that renders Kane's method simpler, and, one may say, more elegant, than Gibbs'.

Another claim made by Desloge is that Kane's method depends explicitly on Cartesian coordinates. On the contrary, the method permits the use of any generalized coordinates whatsoever. Moreover, involving generalized speeds in a central way, it permits one to effect a distinct and often advantageous separation of dynamical and kinematical considerations. The superiority of Kane's method to Gibbs' will become readily apparent to Desloge, as it already has to many dynamicists, when he applies both methods to the formulation of explicit equations of motion for a complex multibody system of the kind frequently encountered in modern engineering practice.

Reference

[1]Desloge, E. A., "Relationship Between Kane's Equations and the Gibbs-Appell Equations," *Journal of Guidance, Control, and Dynamics*, Vol. 10, Jan.-Feb. 1987, pp. 120-122.

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